Advanced Algebra Unit 5 Notes Exponent & Logarithms

Review Exponent Properties

<table>
<tr><td>Zero Exponent</td><td>$b^0 =$
$5^0 =$ $x^0 =$ $(-17)^0 =$
A number raised to the 0 power is always 1.</td><td>Negative Exponent</td><td>$b^{-n} =$ $\frac{1}{a^{-m}} =$
$3^{-4} =$ $x^{-11} =$ $\frac{1}{4^{-2}} =$ $\frac{1}{x^{-12}} =$
A negative exponent causes the number to be re-written as the reciprocal of the original number and the exponent becomes a positive.</td></tr>
<tr><td>Products Property</td><td>$b^n b^m =$
$5^3 5^4 =$ $x^6 x^{-2} =$
When multiplying powers with the same base, keep the base, and add the exponents.</td><td>Power Property</td><td>$(b^n)^m =$
$(x^3)^4 =$ $(4^2)^5 =$
When taking a power to a power, keep the base, multiply the exponents.</td></tr>
<tr><td>Quotient Property</td><td>$\frac{b^n}{b^m} =$
$\frac{5^6}{5^2} =$ $\frac{x^{21}}{x^8} =$
When dividing powers with the same base, keep the base, subtract the exponents.</td><td>Distributive Property</td><td>$(b^n c^m)^p =$
$\left(\frac{b^n}{c^m}\right)^p =$
$(3t^2x^3)^4 =$ $\left(\frac{6x^3}{5y^7}\right)^2 =$
When taking multiply powers to a power, keep the base, and multiply the outside exponent to each of the insides ones.</td></tr>
</table>

$7^2 = 49$	$\log_{12} 1 = 0$	$3^x = 81$
$\log_3 x = 243$	$\log_4 0.0625 = -2$	$5^x = 125$

Quick Write: How do you say $\log_3 x = 4$?

Quick Write: How do you convert between log & exponent form?

Summary:

Introduction to Logarithms

Essential Question: How are logs related to exponents?

Logarithmic Functions: Logs are the **inverse** of Exponential Functions.

Reminder: Inverse functions are ones whose inputs and output as switched, reflecting them over the line $y = x$

Exponent Form	Logarithmic Form
$$a^x = b$$	$$\log_a b = x$$
How do we say it?	How do we saw it?

How to convert to log form?	How to convert to exponent form?
- The base becomes the base of the log - The exponent is what the log equals - The argument become the full-sized number the by log. $a^x = b$ becomes	- The base of the log becomes the base - The full sized number becomes the argument - The solution becomes the exponent. $\log_a b = x$ becomes

Convert to logarithm form.		Convert to exponent form.	
$2^6 = 64$	$4^1 = 4$	$\log_{10} 100 = 2$	$\log_7 49 = 2$
$5^0 = 1$	$5^{-2} = 0.04$	$\log_8 0.125 = -1$	$\log_5 5 = 1$

Special Properties of Logs

Logarithm of base b	Exponential Form		
$\log_b b = 1$	$b^1 = b$	$\log_{10} 10 =$	$10^1 =$
Logarithm of 1 $\log_b 1 = 0$	Exponential Form $b^0 = 1$	$\log_{10} 1 =$	$10^0 =$

Properties of Logarithms

Condense to a single logarithm. Simplify if possible.		
$\log_2 16 - \log_2 2$	$\log_8 4 + \log_8 16$	$\log_6 9 + \log_6 4$
$6\log_5 11 + 6\log_5 6$	$30\log_{10} x - 5\log_{10} y$	$\log_2 x + \log_2 y + 3\log_2 z$
Expand each logarithm.		
$\log_9 \frac{5}{7^3}$	$\log_8\left(7\sqrt{11 \cdot 6}\right)$	$\log_5 \left(\frac{x}{y^6}\right)^3$
$\log_4(u^4 v^2)$	Simplify. $\log_2 16^3$	Simplify. $\log(100)^{0.1}$
Simplify. $\log_3 3^{7+3}$	Simplify. $3^{\log_3 4.52}$	Simplify. $\log_9 6561$

Quick Write: What logs properties are like exponent properties?

Quick Write: How are radicals an extension of the power property?

Summary:

Properties of Logarithms

Essential Question:

	Rule	Examples Write as a single log and/or simplify.
Products Property of Logs	$$\log_b mn = \log_b m + \log_b n$$ Multiplication in the argument can be changed to addition of two logs with the same base, each with 1 of the factors.	$\log_4 2 + \log_4 32$ $\log_5 625 + \log_5 25$
Quotient Property of Logs	$$\log_b \frac{m}{n} = \log_b m - \log_b n$$ Division in the argument can be changed to subtraction of two logs with the same base, the numerator in the first log and the denominator in the back.	$\log_3 54 - \log_3 2$ $\log_{\frac{1}{3}} 1 - \log_{\frac{1}{3}} 9$
Power Properties of Logs	$$\log_b a^p = p \log_b a$$ A logarithm raised to a power is the same as that log times the power.	$\log_4 64^3$ $\log_2 \left(\frac{1}{2}\right)^5$
Inverse Properties of Logs	$$\log_b b^x = x$$ $$b^{\log_b x} = x$$	$\log_4 4^{3x+1}$ $2^{\log_2 8x}$
Change of Base formula	$$\log_b x = \frac{\log_a x}{\log_a b}$$ Your calculator is always base 10, so we need this rule to type logs in to our calculator.	Set it up & use the calculator $\log_4 8$ Pick a base that allows you to do it mentally. $\log_4 8$

Solving Basic Logarithmic Equations

Essential Question:

Solving for the Argument

We use this when we have 1 log that is equal to a constant and the variable is in the argument.

1) Get the log alone on one side of the equal sign (remember the log includes the base and the full-sized number)
   - Apply the log properties
   - Use inverse operations.
2) Convert to exponent form
3) Simplify (rase the base to the power)
4) Solve

$\log_x(x-5)=2$	$\log 45x - \log 3 = 1$
$\log_5(x+4)+\log_5 8 = 3$	$\log_4 x^2 = 7$

Logs with matching bases

We use this when there is a log on each side of the equation and the bases are the same.

1) Apply log properties to get 1 single log on each side of the equation.
2) Double check that the bases are the same number
3) Set the arguments equal to each other.
4) Solve.

Reminder: Equals means "the same as**," when we are solving an equation we are saying what value of the variable will make the two sides the same**

$\log_3(2k+4)=\log_3 4$	$\log_{14}(-5x-4)=\log_{14} -x$
$\log_6(x+5)-\log_6 2=\log_6 35$	$\log -5x - \log 5 = \log 75$

Quick Write: What part of the log is the "argument"

Quick Write: How do you know when to use each strategy?

Summary:

Solving Basic Exponential Equations

Essential Question:

Solving for the Exponent

We use this when there is only 1 exponent and it is the variable.

1) Get the base and exponent alone on one side of the equal sign.
   - Apply power properties
   - Use inverse operations
2) Convert to log form.
3) Simplify
4) Solve

$3^x = 70$	$5^{x-2} = 3125$
$13^{-7x} - 6 = 30$	$-6 \cdot 12^{-8n} = -36$

Exponential Equations with matching bases.

We use this when there is an exponent on each side of the equation and the bases are the same or we can make them the same through exponent rules.

1) Apply exponent properties to get one base and exponent on each side. The bases must match.
2) Double check that the bases are the same number.
3) Set the exponent equal to each other.
4) Solve

Reminder: Equals means "the same as**," when we are solving an equation we are saying what value of the variable will make the two sides the same**

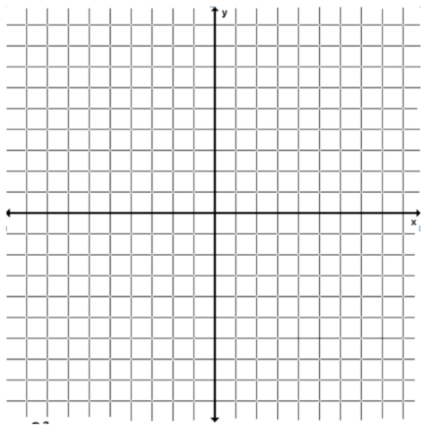
$3^{-b-1} = 3^{3-2b}$	$7^{-2x} \cdot 7^x = 7^{-3x}$
$8^{3x} = 64^{x+3}$	$125^{2x} = 25^{3x}$

Quick Write: If the bases don't match can you get them to match? How?

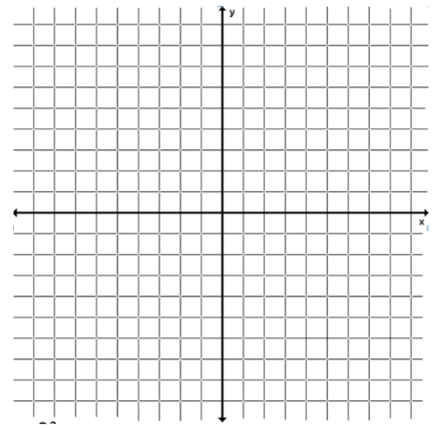
Summary:

Graphing Exponential Functions

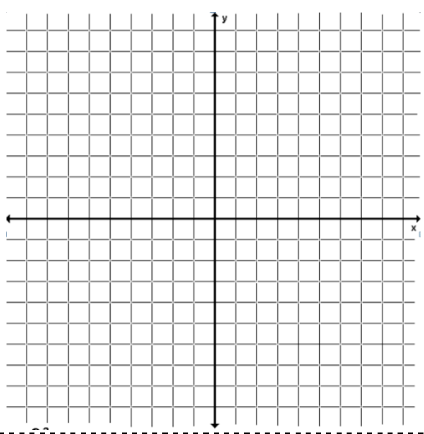
$f(x) = 3 \cdot 2^x$



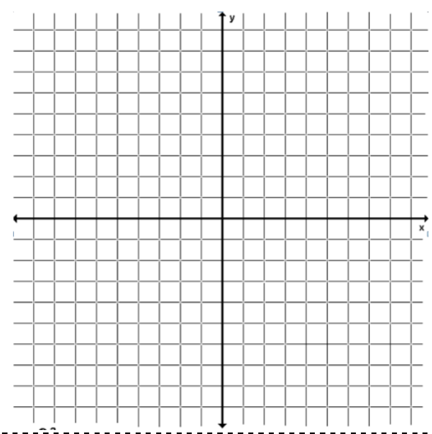
$f(x) = -1 \cdot 3^x + 2$



$f(x) = \left(\frac{1}{2}\right)^x$



$f(x) = 3\left(\frac{1}{2}\right)^x$



Quick Write: What do negative exponents do to the base?

Quick Write: What part of the equation changes the horizontal asymptote?

Summary:

Graphing Exponential Functions

Essential Question: Where are the asymptotes and how do they help us graph?

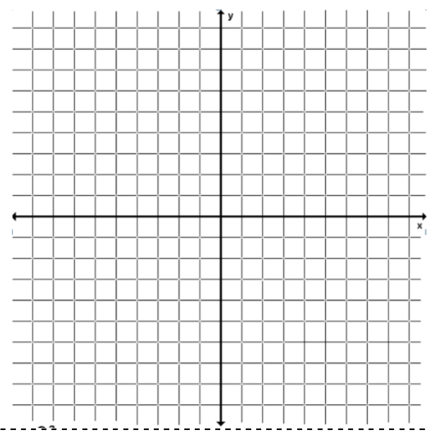
Negative Exponents:	Zero Exponent:
$2^{-3} =$ $5^{-1} =$ $\frac{1}{3^{-2}} =$	$2^0 =$ $5^0 =$ $\frac{1}{3^0} =$

Steps

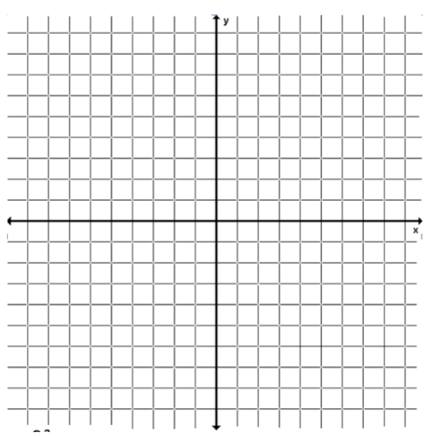
$$f(x) = ab^{x-h} + k \text{ where } b > 0, b \neq 1$$

1) Write down any anticipated shifts
2) Draw in your asymptote (Use k to help)
3) Make a table
4) State the domain & range.

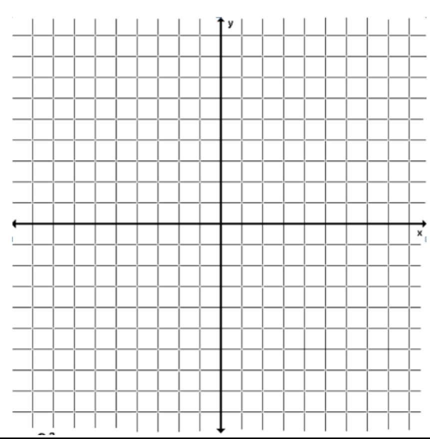
$f(x) = 2^x$



$f(x) = 2^x - 3$



$f(x) = 2^{x-3}$

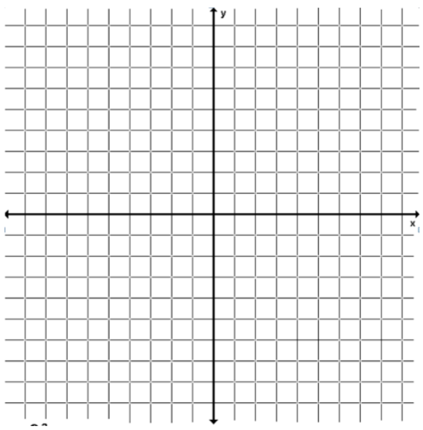


Domain: All possible *x* values
Start at the left of the graph and move to the right. Where does the graph start and stop? Are there any asymptotes?

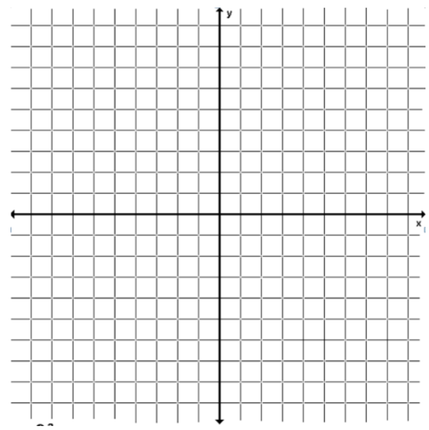
Range: All possible y-values
Start at the bottom of the graph, move to the top. Where does the graph start and stop? Are there any asymptotes?

Graphing Log Functions

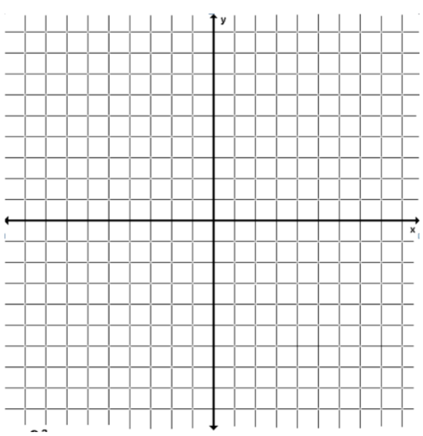
$y = \log_3 x$



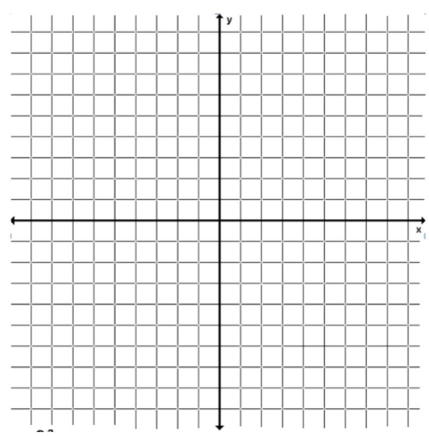
$y = 2\log_3(x + 4)$



$y = \log_4(x + 2) - 6$



$y = 3\log_2(x - 1) + 4$



Quick Write: How is graphing a log like graphing an exponential function?

Quick Write: How are the transformations of the logs like other transformations we have seen before?

Summary:

Graphing Log Functions

Essential Question: How do we use exponential functions to help us graph logs?

Converting from Log to Exponential form.

$y = \log_4 x$ $y = \log_2 x$

What are inverse operations?

Steps to Graphing Basic Logs

Remember: Logarithms are the inverse of exponential functions, which means their inputs and out puts are switched. We use this to graph the logs.

1) Convert the equation in to exponential form.

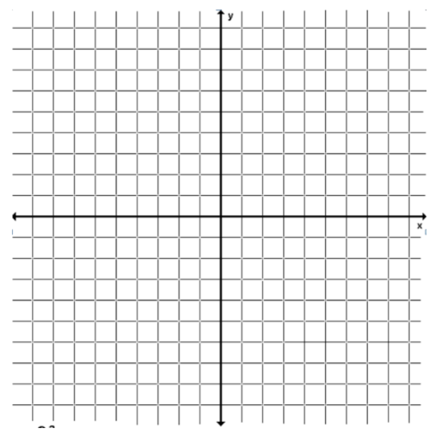
2) Set up a table but pick the ***y*-values** and use those are the inputs.

3) Fill in the ***x*-value**s with your outputs

4) Plot the points.

** Logs have a vertical asymptote at $x = 0$**

$y = \log_2 x$



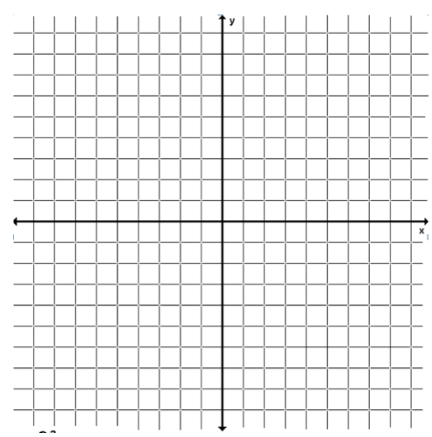
Steps to Graphing Logs with Transformations

1) Make a table for the basic log function (directions above)

2) Perform the indicated transformations.
   - This can be on a table, or by moving points on the graph.
   - Be sure to more the asymptote.

$$\boldsymbol{f(x) = a\log_b(x-h)+k}$$

$y = \log_2(x-3)-4$



Domain: All possible x values

Start at the left of the graph and move to the right. Where does the graph start and stop? Are there any asymptotes?

Range: All possible y-values

Start at the bottom of the graph, move to the top. Where does the graph start and stop? Are there any asymptotes?

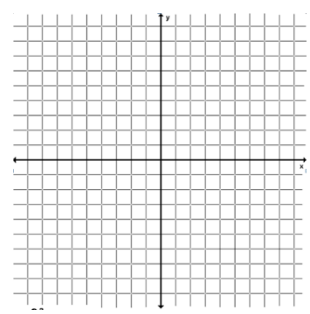
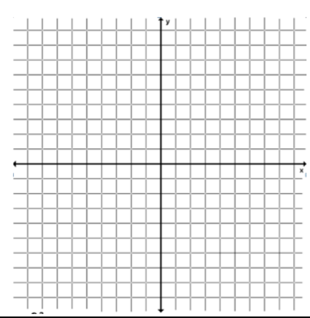
Exponential Growth and Decay

An acidophilus culture containing 150 bacteria doubles in population every hour. Predict the number of bacteria after 12 hours.

1. Write a function representing the bacteria population for every hour that passes.
2. Graph the function.
3. Use the graph to predict the number of bacteria after 12 hours.

A new softball dropped onto a hard surface from a height of 25 inches rebounds to about -2- the height on each successive bounce. Write a function representing the rebound height for each bounce. Graph the function.
After how many bounces would a new softball rebound less than 1 inch?

The Natural Base e.

What is e? Given the equation $f(n) = \left(1 + \frac{1}{n}\right)^n$ (to represent interest) As n gets large interest is continuously compounded, as n gets larger it approaches 2.7182818… So $e = 2.7182818$, it is an irrational constant.	**Graph:** $f(x) = e^x$ 
What is a Natural Log? It is a logarithm with base *e*, it is called the natural log and we write it as ln. $\log_e x = \ln x$	**Graph:** $f(x) = \ln x$ 

Summary:

Exponential Growth and Decay

Essential Question:

Exponential Functions	Exponential Growth and Decay
$f(x) = b^x$ where $b > 0, b \neq 1$	$\boldsymbol{A(t) = a(1 \pm r)^t}$

Does the function show growth or decay then make a table.

$f(x) = 32(0.5^x)$

$f(x) = 0.5(1.2^x)$

$f(x) = 0.4\left(\frac{3}{4}\right)^x$

Tony purchased a rare 1959 Gibson Les Paul guitar in 2000 for $12,000. Experts estimate that its value will increase by 14% per year. Use a graph to find when the value of the guitar will be $60,000.

The value of a truck bought new for $28,000 decreases 9.5% each year. Write an exponential function, and graph the function. Use the graph to predict when the value will fall to $5000.